Effect of Feeder Cable's Phase Tolerance on the First Sidelobe Level of Base Station Antenna

Yejun He[†‡], Zhengzheng Pan[†], Jie Yang[†], Guiyuan Sun[†] and Manos M. Tentzeris[‡]
[†] College of Information Engineering, Shenzhen University, 518060, China
[‡] School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA, 30332, USA

Email: {heyejun, panzz89, yangjie8925, sunguiyuan66}@126.com, etentze@ece.gatech.edu

***Abstract*—The sidelobe level of a base station antenna is one of the important parameters to describe the performance of an antenna array. Given a required value of the sidelobe level, we can obtain a set of initial phases, and then further get a set of cable lengths. However, a tolerance (or error range) associated with manufacturing techniques will introduce an error in each cable length, thereby influencing the sidelobe level. This paper uses the knowledge of probability and mathematical statistics to make a statistical analysis for the reliability of the first sidelobe of the antenna array based on Monte Carlo simulations. We also obtain a distribution curve of reliabilities of the first sidelobe versus phase tolerances, which can bring great convenience for practical applications.**

Keywords-phase tolerance; Monte Carlo; MATLAB; first sidelobe

I. Introduction

Sidelobes are the radiation lobes in any direction of an antenna pattern except the main lobe. They reflect the spatial distribution of the antenna radiated electromagnetic energy and show the spatial selectivity of receiving electromagnetic energy [1]. In recent years, low sidelobe antenna arrays have received widespread attention [2], [3]. Theoretically, we can design antenna arrays with arbitrary sidelobe levels. However, array errors and various other defects will affect the sidelobe in practice. For example, Allen et al [4] analyzed the effect of the amplitude-phase error of an array element on the sidelobe level, gain and beam steering of an antenna array. J. K. Hsiao [5] studied the relationship between the sidelobe level and random error of an antenna array. J. Ruze [6] studied the effect of random errors due to excitation current on the array radiation lobe. E. Schanda [7] showed that random amplitude errors with zero mean have equal influence as phase errors. Ding Yu et al [8] studied the effect of system phase errors on the results of planar near-field measurements on ultra-low sidelobe antennas. Guangzhi Xiang [9] analysed the errors of an ultra-low sidelobe antennas in S band.

However, the amplitude and phase of antenna excitation signals obviously affect the antenna pattern. In order to make the antenna pattern meet requirements, we have a certain standard of the amplitude and phase tolerance when buying cables. The smaller the tolerance is , the better the performance of antenna pattern is. It is difficult for manufacturers to achieve high precision, and low precision cannot meet the performance of the antenna pattern. So we need to know the effect of feeder cable's phase tolerance on antenna patterns.

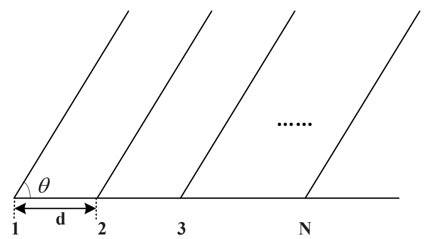


Fig. 1. Array antenna coordinate diagram.

This paper mainly takes the phase tolerance of cables as an example and makes a detailed analysis for the sidelobe level of an array antenna based on MATLAB software platform and the Monte Carlo method. Finally we obtain a distribution curve of reliability of the first sidelobe versus phase tolerance.

II. Theoretical Foundation

Assume that the antenna array is composed of N radiating elements, and the spacial distance between two adjacent elements is d, as illustrated in Fig. 1. The array factor is as follows [10], [11]:

$$F(\theta) = \sum_{n=1}^{N} f_n(\theta) * I_n * exp\{j(k * Z_n * \cos\theta + \phi_n)\} \quad (1)$$

where $f_n(\theta)$ means an element pattern; I_n stands for the amplitude of the nth element (i.e. the amplitude of cables); $k = 2\pi/\lambda$ (λ=wavelength); $Z_n = (n-1)d$ is the distance between the original point and the nth element, d is the distance between two adjacent elements; θ presents the beam steering angle; ϕ_n stands for the phase of the nth element (i.e. the phase of cables). We can obtain the amplitude of the array factor from (1):

$$TempF = |F(\theta)| \quad (2)$$

The normalized amplitude lobe pattern is obtained by normalizing the maximum amplitude, which is also the dimensionless function of angles. Its maximum value is 1. Therefore, the normalized amplitude lobe pattern formula should be:

$$TempFN = TempF/(TempF)_{max} \quad (3)$$

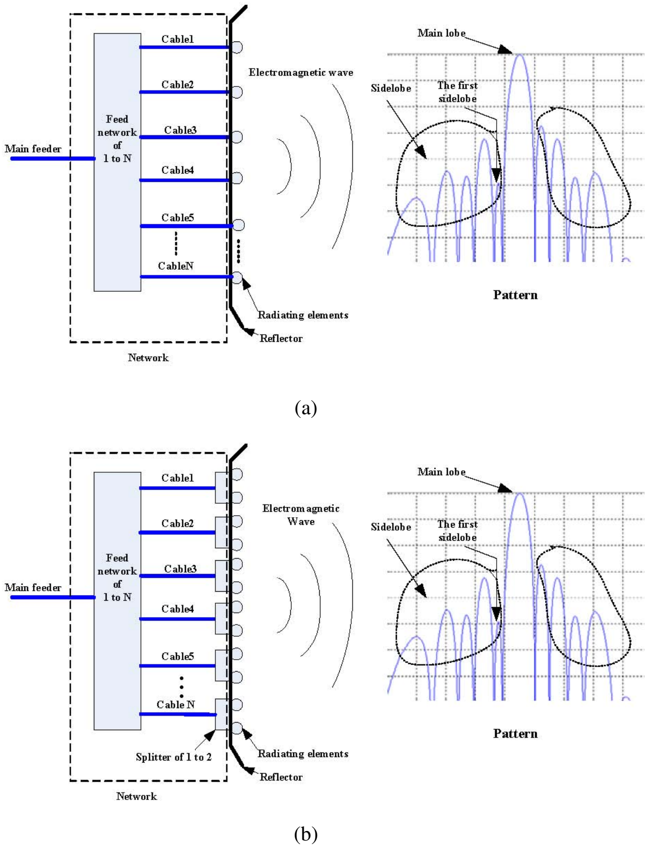


Fig. 2. Working principle diagram. (a) Feed network without power dividers. (b) Feed network with power dividers.

where $(TempF)_{max}$ is the maximum of the amplitude of array factor.

The normalized power lobe pattern formula in dB becomes

$$E = 20 * log_{10}(TempFN) \tag{4}$$

III. Working principle

In general, an antenna array is composed of three components− feed network, reflector and radiating elements. Fig. 2 is a working principle diagram of a single-polarized antenna which consists of N radiating elements as shown in Fig. 2(a) or $2N$ radiating elements as shown in Fig. 2(b). When an input signal is transmitted to the main feeder, it will go through 1-dividing-N feed network, and then allocates N signals to corresponding N radiating elements or N signals to corresponding $2N$ radiating elements. Finally these radiation elements will radiate signal in all directions in the form of electromagnetic waves. Thus a total pattern can be formed in the distance (it can be calculated from formulae (1) to (4)). This pattern is used to examine antenna performance. Usually we require a higher main-lobe level and lower sidelobe levels (also known as minor lobes). In practical applications we often evaluate the sidelobe height in terms of the first sidelobe.

The main factors influencing the first sidelobe performance of an antenna are the amplitude and phase of radiating elements, which are mainly controlled by the feed network. A feed network usually consists of PCB network, power dividers (if applicable) and cables. After confirmation of the antenna solution, the PCB network and power dividers have been fixed, therefore the only thing that can adjust the amplitude and phase of radiating elements is varying the cable length. That is to say, different cable lengths correspond to different amplitudes and phases. Cable lengths are assumed to follow the linear distribution with amplitudes and phases of radiating elements in theory.

Hence, according to inverse method from (4) to (1), given the required value of the first sidelobe level, we can obtain a group of corresponding initial phases. Then the phase of each cable is calculated by substracting the phase of the feed network (PCB network) and each power divider (if applicable) from each initial phase. Further, we obtain each cable length. There will be an error in length introduced by tolerances set during mass production. The error effects the antenna array at different levels. This paper starts from this point, and analyzes the effect of feeder cable's phase tolerance on the first sidelobe level of base station antenna based on MATLAB software platform and the Monte Carlo method.

IV. Monte Carlo Simulation

Monte Carlo simulation method is also called statistical simulation method or stochastic simulation method [12]. It is based on statistical sampling theory and is aided by a computer. After relevant statistical sampling or stochastic simulation for a random variable are used, functions of statistics are estimated and described. Monte Carlo simulation method is a numerical calculation one to be used in an approximate solution to solve engineering problems. It can guarantee probability convergence. So it is widely used in engineering. The theoretical basis of Monte Carlo simulation comes from two basic theorems of probability theory.

A. Law of Large Numbers

Supposed that $X_1, X_2...X_n$ are independent random variables, from the same population, and are identically distributed, which means they have the same mean μ and variance σ^2. For any $\varepsilon > 0$, we have

$$\lim_{n\to\infty} P\{|\frac{1}{n}\sum_{k=1}^{n} X_k - \mu| \geq \varepsilon\} = 0 \tag{5}$$

B. Bernoulli's Theory

Assume that μ_n is number of times that event A occurs in n times test events, and p is the probability of event A occurred in each test. For any positive integer ε, we have

$$\lim_{n\to\infty} P\{|\frac{\mu_n}{n} - p| < \varepsilon\} = 1 \tag{6}$$

Monte Carlo method draws simple samples to do sampling test from the same population [13] [14]. From equation (5) and (6), we know that when n is large enough, the sample mean $\frac{1}{n}\sum_{k=1}^{n} X_k$ is convergent to μ with probability 1 and sample frequency $\frac{\mu_n}{n}$ is convergent to p with probability 1 [15].

Therefore, in theory, this method provides a wide range of applications. When using the Monte Carlo method to solve the probability of the occurrence of a certain event, we can get the probability of the event by applying the sampling test method and the probability is a solution of the problem. Specific steps are: according to the composition of the system and other relative amount of actual parameters of the distribution of X, do random sampling $(X_1, X_2...X_k)$ for X for the first time, and put it into the performance parameters of the expression. Finally, we get the first random values:

$$Y_1 = f(X_1, X_2...X_k) \tag{7}$$

Repeating this operation n times produces n random values Y_1,Y_2,......,Y_n. Thus statistical analysis on Y_1,Y_2,......,Y_n will be carried out, and the probability of the occurrence of Y_n in given conditions can be gained. The larger the sample number n, the more accurate the estimation of the probability distributions during Monte Carlo analysis.

V. MATLAB Simulation Program and Practical Examples analysis

Based on the Monte Carlo simulation, we set up a phase tolerance of D, and randomly extract n group phases in the allowed tolerance range and bring each phase into formula (1), respectively. Then we can get n antenna patterns. By using MATLAB software platform, we read the first sidelobe levels of each pattern. Through statistical calculations, we also get a distribution of the first sidelobe levels. In other words, these first sidelobe levels are obtained in the condition of initial phase tolerances. Further, we calculate a probability where the first sidelobe level meets a required value, which is also called the reliability of the system performance. The flow chart of the MATLAB simulation program is shown in Fig. 3 .

A. Simulated Analysis when feeder cable's phase tolerance is $\pm 4°$

For convenience in simulation and measurement, we take Fig. 2(a) into consideration with seven radiating elements and analyze the relationship between the phase tolerance of cables and reliability of the first sidelobe.

The systematic specific parameters are as follows: the radiation element number is $N = 7$; the spatial distance between two adjacent elements is $d = 280$ mm; the frequency in this system is 880 MHz; the initial values (normalized) of the amplitude of the cable are $[0.685, 0.685, 0.79, 1, 0.77, 0.67, 0.67]$, the amplitude tolerance is 0 dB; the initial values of the phase of the cable are $[-7°, 5°, 8°, 0°, -8°, -5°, 7°]$, the phase tolerance denoted by D is $\pm 4°$. Monte Carlo analysis accuracy is directly related to the sampling times. Sampling times can be revised according to requirements. Assuming 1000 samplings has met the statistical accuracy in this case, then the random sampling frequency for the cable's phase is 1000. According to the above settings, by MATLAB simulation, we obtain 1000 decibel level curves. The horizontal axis represents the independent variable beam steering angle θ, and the vertical axis means the decibel level value (E: Amplitude), as shown in

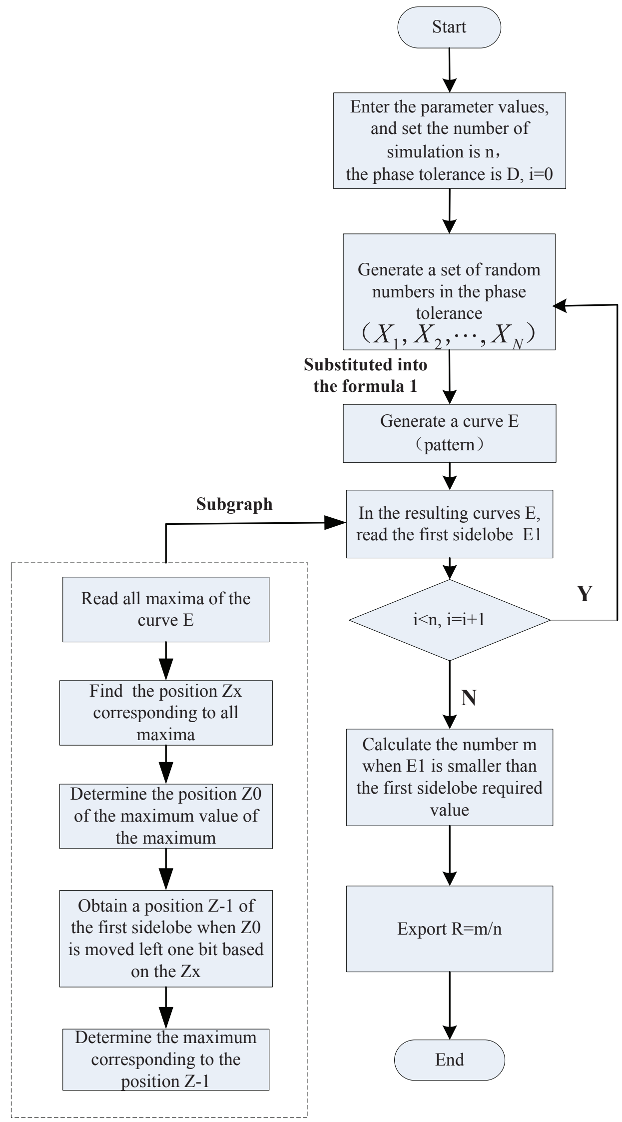


Fig. 3. The flow chart of the MATLAB simulation program

Fig. 4. From Fig. 4, we cannot intuitively judge the distribution of the first sidelobe. So we have to know levels of the 1000 curves of the first sidelobe further. The statistics are shown in Table I when $|D|$ is $4°$.

The corresponding histogram is shown in Fig. 5. In sampling n times, the value of the system performance is m within bounds. Therefore the reliability of the system performance can be expressed as

$$R = \frac{m}{n} \tag{8}$$

Assume the first sidelobe level should be smaller than -24 dB in engineering applications. From TABLE I, the number of times where the first sidelobe level is smaller than -24 dB is 617 during 1000 samples. According to the formula (8) the reliability of the first sidelobe is $R = 617/1000 = 0.617$.

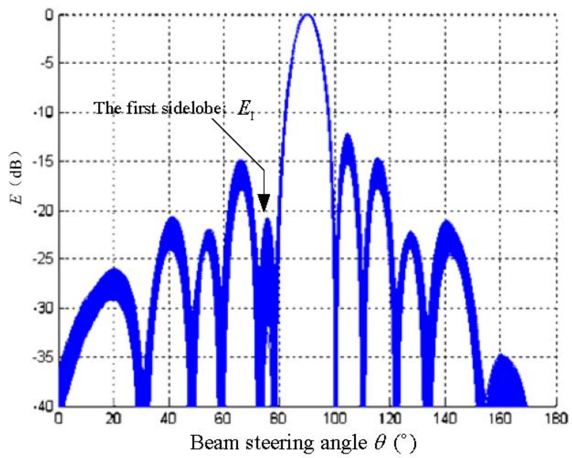


Fig. 4. Simulated antenna pattern for 880 MHz when the phase tolerance is ±4°

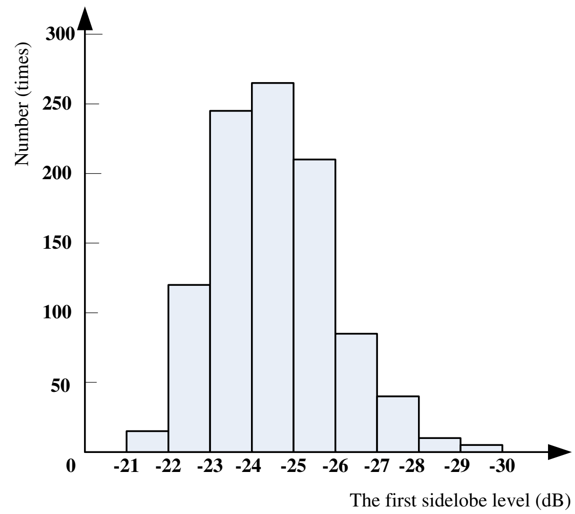


Fig. 5. Histogram

TABLE I
THE FIRST SIDELOBE LEVEL WHEN $|D|$ IS 4°

Intervals	Number of times in the corresponding intervals
(-21dB, -22 dB)	14
(-22 dB,-23 dB)	121
(-23 dB,-24 dB)	248
(-24 dB,-25 dB)	264
(-25 dB,-26 dB)	209
(-26 dB,-27 dB)	86
(-27 dB,-28 dB)	41
(-28 dB,-29 dB)	11
(-29 dB,-30 dB)	6

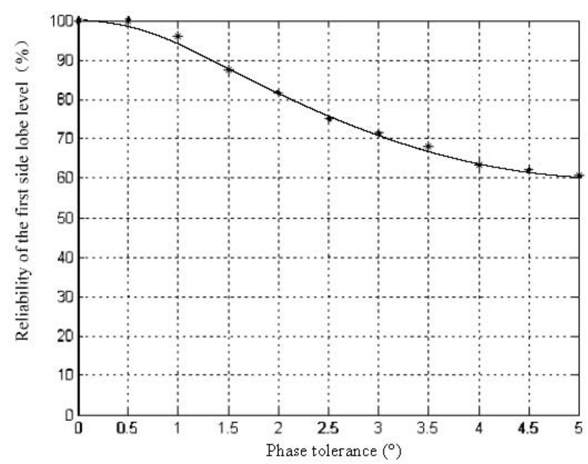


Fig. 6. Reliability of the first sidelobe versus various phase tolerances

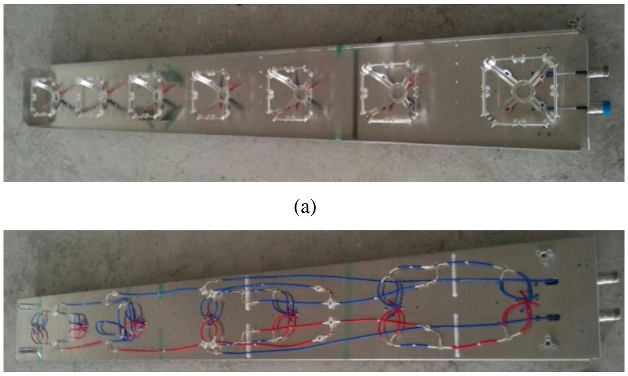
B. Simulated Analysis when feeder cable's phase tolerance is in the range [0°, 5°]

Assume the absolute value of the phase tolerance $|D|$ is [0°, 5°], and the step size between the tolerances is 0.5. By circular operation for program similar to the phase tolerance of ±4°, we obtained a curve about reliability of the first sidelobe versus phase tolerance as shown in Fig. 6. In Fig. 6, the horizontal axis represents the phase tolerance, and the vertical axis is the reliability of the first sidelobe (that is probability). The larger the phase tolerance, the lower the reliability of the first sidelobe. So we should choose the appropriate phase tolerance on demand to meet the requirements of the first sidelobe level.

C. Measured Analysis when feeder cable's phase tolerance is 0° and 1°

In our experimentation, 30 base station antennas with 7 elements are used as shown in Fig. 7 and 30 sets of cables (each set of cable consists of 7) are cut due to cable costs and experimental places. Firstly, we assemble one set of cable in one base station antenna. If the perfect lengths of seven cables (i.e the phase tolerance is 0°) are 176 mm, 217 mm, 515 mm, 756 mm, 732 mm, 530 mm and 480 mm, respectively, then the measured E-plane pattern for the 790-960 MHz frequency range is obtained as shown in Fig. 8, where the first sidelobe level is -24.5 dB when the center frequency is 880 MHz.

In practical applications, we hope the phase tolerance is



(a)

(b)

Fig. 7. Practical base station antenna. (a) Front view. (b) Back view.

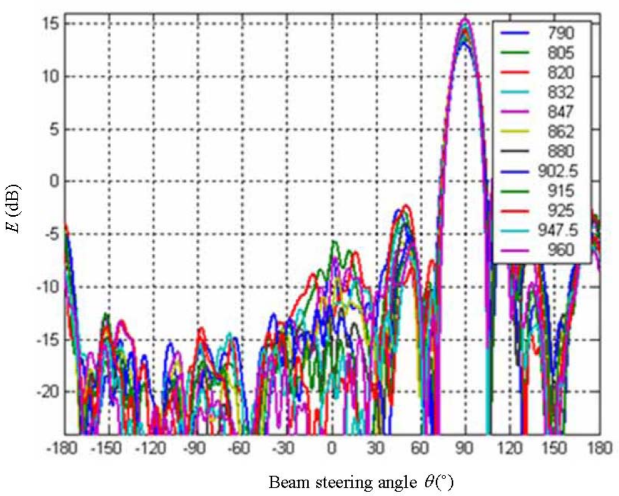


Fig. 8. Measured antenna pattern for the 790-960 MHz frequency range when the phase tolerance is 0°

small enough. Assume the phase tolerance is 1°, where the phase tolerance of 1° is equal to the length tolerance of 3 mm. By repetitious assembly and experimentation (30 sets of cables are connected to 30 antennas randomly), we obtain some statistics of the first sidelobe level at 880 MHz as follows: -24.2 dB, -24 dB, -24.8 dB, -25 dB, -24.5 dB, -24.7 dB, -23.7 dB, -25 dB, -25 dB, -24.3 dB, -24.4 dB, -24.5 dB, -25.5 dB, -24.6 dB, -24.3 dB, -25 dB, -24.4 dB, -24.6 dB, -24 dB, -24 dB, -25 dB, -25.2 dB, -24.9 dB, -24.3 dB, -23.8 dB, -24.1 dB, -24 dB, -25.6 dB, -25 dB, -24.5 dB. Here, the measured probability, which is defined as the first sidelobe level measured is not bigger than -24 dB, is $28/30 = 93.3\%$. However, the simulated probability from previous 1000 times sampling simulation, which is defined as the first sidelobe level simulated is not bigger than -24 dB, is $950/1000 = 95\%$. The difference between the measured probability and the simulated probability is resulted from less measured times comparing to simulated times. But the trend is approaching the simulated case and this method provides an important reference.

VI. Conclusion

By simulation with MATLAB, we obtain the relationship between the hase tolerance and the reliability of the first sidelobe level prior to knowing any distribution of these tolerances. In the past, researchers think that the amplitude and phase tolerances obey some distributions. Secondly, this Matlab simulation platform provides flexibility to study effect feeder cable's phase tolerance on the first sidelobe level. We can modify the program and get a distribution curve of reliabilities of the the first sidelobe in different elements or different frequencies to use in various antenna systems. All in all, the relationship between reliabilities of the first sidelobe and feeder cable's phase tolerances provides a powerful guidance for mass production of antennas.

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References

[1] Rone-Hwa Chou, Chan-Shan Yang, Ci-Ling Pan, "Effects of pump pulse propagation and spatial distribution of bias fields on terahertz generation from photoconductive antennas," *Journal of Applied Physics*, vol. 114, no. 4, pp. 043108-043108-7, July 2013.

[2] S. I. Krich, I. Weiner, C. J Prust, "Low sidelobe antenna patterns with failed elements," *2012 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 2581-2584, 2012.

[3] A. Falahati,M. NaghshvarianJahromi, R.M. Edwards, "Wideband Fan-Beam Low-Sidelobe Array Antenna Using Grounded Reflector for DECT, 3G, and Ultra-Wideband Wireless Applications," *IEEE Transactions on Antennas and Propagation*, vol. 61, no. 2, pp. 700-706, 2013.

[4] J. L. Allen et al, " The theory of array antennas with emphasis on radar applications," Tech. Rep.323, MIT Lincoln Lab, Lexington, Massachusetts, July, 1963.

[5] J. K. Hsiao. *Effects of errors on the sidelobe level of a low sidelobe array antenna.* Washington: Naval Research laboratory, 1981.

[6] J. Ruze, "The effect of aperture errors on the antenna radiation pattern," *Nuovo Cimento Suppl.*, vol. 9, no. 3, pp. 364-380, 1952.

[7] E. Schanda, "The effect of random amplitude and phase errors of continuous apertures," *IEEE Transactions on Antennas and Propagation*, vol. 15, no. 3, pp. 471-473, 1967.

[8] Ding Yu, Lin Yang, De-min Fu, Qi-zhong Liu, "Analysis and simulation of system phase errors in planar near-field measurements on ultra-low sidelobe antennas," *2010 IEEE International Conference on Ultra-Wideband (ICUWB)*, pp. 1-4, 2010.

[9] Guangzhi Xiang, "Analysis on the errors of an ultra-low sidelobe array antenna," *Modern Radar*, no. 6, pp. 39-48, 1996.

[10] Yejun He, Liangqi Gui and Xia Li, *Smart Antennas (Matlab edition).* (in Chinese), Publishing House of Electronics Industry, Beijing, Dec. 2009.

[11] Yejun He and Liangqi Gui, *Field and Wave Electromagnetics (Second Edition)* (in Chinese), Tsinghua University Press, Beijing, 2013.

[12] N. T. Thomopoulos. Essentials of of Monte Carlo Simulations: Statistical Methods for Building Simulation Models. Springer, 2013.

[13] Brian Korver. *The monte carlo method and software reliability theory*. 1994.

[14] Lucheng Pei, Zhongqi Wang. *Monte Carlo method and its application*. China Ocean Press, 1998.

[15] Jay L. Devore. *Probability and statistics for engineering and the sciences(6th ed)*. Thomson-Brooks/Cole, Belmont, CA, 2004.